JEE Main 2023

(January Attempt)

24 JANUARY 2023

SHIFT – I

MARKS : 100 **60 min.**

SCQ

1. Let Ω be the sample space and $A \subseteq \Omega$ be an event.

Given below are two statements

(S_1) : If $P(A) = 0$, then $A = \phi$

(S_2) : If $P(A) = 1$, then $A = \Omega$

Then,

(A) Only (S_2) is true
(B) Both (S_1) and (S_2) are true
(C) Only (S_1) is true
(D) Both (S_1) and (S_2) are false

2. The distance of the point (7, –3, –4) from the plane passing through the points (2, –3, 1), (–1, 1, –2) and (3, –4, +2) is

(A) 4
(B) $4\sqrt{2}$
(C) 5
(D) $5\sqrt{2}$

3. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right)+\sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$ is equal to

(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{6}$
(C) $\frac{\pi}{2}$
(D) $\frac{\pi}{3}$

4. Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at the points A and B. Then, the mid points of such line segment AB lies on a parabola with the

Ⓐ length of latus rectum $\frac{2}{3}$

Ⓑ directrix $4x = -3$

Ⓒ length of latus rectum $\frac{3}{2}$

Ⓓ directrix $4x = 3$

5. The distance of the point (–1, 9, –16) from the plane $2x + 3y - z = 5$ measured parallel to the line $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ is

Ⓐ $20\sqrt{2}$

Ⓑ 26

Ⓒ 31

Ⓓ $13\sqrt{2}$

6. Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ, respectively such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then, $\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)}$ is equal to

Ⓐ $\frac{5}{2}$

Ⓑ 3

Ⓒ 4

Ⓓ 2

7. The equation $x^2 - 4x + [x] + 3 = x[x]$, where [x] denotes the greatest integer function, has

Ⓐ a unique solution in $(-\infty, \infty)$

Ⓑ no solution

Ⓒ a unique solution in $(-\infty, 1)$

Ⓓ exactly two solutions in $(-\infty, \infty)$

8. The value of $\sum_{r=0}^{22} {}^{22}C_r \, {}^{23}C_r$ is

Ⓐ ${}^{45}C_{23}$

Ⓑ ${}^{44}C_{23}$

Ⓒ ${}^{45}C_{22}$

Ⓓ ${}^{45}C_{24}$

9. Let α be a root of the equation $(a - c)x^2 + (b - a)x + (c - b) = 0$,

where a, b, c are distinct real numbers such that the matrix $\begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$ is singular.

Then, the value of $\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)}$ is

(A) 6 (B) 9 (C) 12 (D) 3

10. $\lim_{t \to 0}\left(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + \ldots + n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$ is equal to

(A) $\frac{n(n+1)}{2}$ (B) n (C) $n^2 + n$ (D) n^2

11. Let, $p, q \in R$ and $(1-\sqrt{3}i)^{200} = 2^{199}(p+iq), i = \sqrt{-1}$.

Then, $p + q + q^2$ and $p - q + q^2$ are roots of the equation.

(A) $x^2 - 4x + 1 = 0$ (B) $x^2 - 4x - 1 = 0$ (C) $x^2 + 4x - 1 = 0$ (D) $x^2 + 4x + 1 = 0$

12. For three positive integers $p, q, r, x^{pq^2} = y^{qr} = z^{p^2 r}$ and $r = pq + 1$ such that 3, $3 \log_y x$, $3 \log_z y$, $7 \log_x z$ are in AP with common difference 1/2. Then, r–p–q is equal to

(A) 12 (B) 2 (C) 6 (D) –6

13. The relation $R = \{(a, b) : \gcd(a, b) = 1, 2a \neq b, a, b \in Z\}$ is

(A) Neither symmetric nor transitive (B) reflexive but not symmetric

(C) symmetric but not transitive (D) transitive but not reflexive

14. If A and B are two non-zero n × n matrics such that $A^2 + B = A^2B$, then

(A) $A^2B = BA^2$ (B) $A^2B = I$ (C) $A^2 = I$ or $B = I$ (D) $AB = I$

15. Let $f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$ Then at $x=0$

Ⓐ f and f′ both are continuous

Ⓑ f is continuous but f′ is not continuous

Ⓒ f is continuous but not differentiable

Ⓓ f′ is continuous but not differentiable

16. Let $u=\hat{i}-\hat{j}-2\hat{k},\ v=2\hat{i}+\hat{j}-\hat{k},\ v\cdot w=2$ and $v\times w=u+\lambda v$. Then, $u\cdot w$ is equal to

Ⓐ 1

Ⓑ $-\frac{2}{3}$

Ⓒ $\frac{3}{2}$

Ⓓ 2

17. Let $y=y(x)$ be the solution of the differential equation $x^3dy+(xy-1)dx=0, x>0, y\left(\frac{1}{2}\right)=3-e$. Then, $y(1)$ is equal to

Ⓐ e

Ⓑ 2 – e

Ⓒ 1

Ⓓ 3

18. The area enclosed by the curves $y^2+4x=4$ and $y-2x=2$ is

Ⓐ $\frac{25}{3}$

Ⓑ $\frac{23}{3}$

Ⓒ 9

Ⓓ $\frac{22}{3}$

19. The compound statement $(\sim P\wedge Q))\vee((\sim P)\wedge\sim Q)\Rightarrow((\sim P)\wedge(\sim Q))$ is equivalent to

Ⓐ $((\sim P)\vee Q)\wedge((\sim Q)\vee P)$

Ⓑ $(\sim Q)\vee P$

Ⓒ $(\sim P)\vee Q$

Ⓓ $((\sim P)\vee Q)\wedge(\sim Q)$

20. Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations

$x+y+z=1$

$2x+Ny+2z=2$

$3x+3y+Nz=3$

has unique solution is $\frac{k}{6}$, then the sum of value of k and all possible values of N is

Ⓐ 21

Ⓑ 18

Ⓒ 19

Ⓓ 20

INTEGER

21. Suppose, $\sum_{r=0}^{2023} r^2\ {}^{2023}C_r = 2023 \times a \times 2^{2022}$. Then, the value of α is ....

22. The value of $\frac{8}{\pi}\int_0^{\pi/2} \frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}} dx$ is ....

23. Let C be the largest circle centred at (2, 0) and inscribed in the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$. If $(1, \alpha)$ lies on C, then 10 α^2 is equal to ....

24. The shortest distance between the lines $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-6}{2}$ and $\frac{x-6}{3}=\frac{1-y}{2}=\frac{z+8}{0}$ is equal to

25. The number of 9-digits numbers, that can be formed using all the digits of the number 123412341, so that the even digits occupy only even places, is ....

26. The value of $12\int_0^3 |x^2-3x+2| dx$ is ....

27. A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is ....

28. Let $\lambda \in R$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then, the largest element in the set $S = \{x + \lambda : x \text{ is an integer solution of E}\}$ is ....

29. Let a tangent to the curve $9x^2 + 16y^2 = 144$ intersect the co-ordinate axes at the points A and B. Then, the minimum length of the line segment AB is ....

30. The 4th term of GP is 500 and its common ratio is $\frac{1}{m}, m \in N$. Let S_n denotes the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is ....

ANSWERS

SCQ

1. Ⓑ	4. Ⓓ	7. Ⓐ	10. Ⓑ	13. Ⓐ	16. Ⓐ	19. Ⓐ
2. Ⓓ	5. Ⓑ	8. Ⓐ	11. Ⓐ	14. Ⓐ	17. Ⓒ	20. Ⓓ
3. Ⓓ	6. Ⓑ	9. Ⓓ	12. Ⓑ	15. Ⓑ	18. Ⓒ	

INTEGER

21. 1012	23. 118	25. 60	27. 546	29. 7
22. 2	24. 14	26. 22	28. 5	30. 12